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Reply To:
J020

24 Sep 1996

MEMORANDUM FOR J535

Subject: Review of Paper for Publication

1. J020 has reviewed your paper, *A Target Selection Tool for NETWORK INTERDICTION,* and deems it appropriate for public release.

2. If you have any questions, please contact me at 4-1068/5963.

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DTIC QUALITY INSPECTED 1

A Target Selection Tool for NETWORK INTERDICTION

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ABSTRACT

In an era of aggressive strategic force reductions it has become increasingly important to plan available weapons in the most effective manner possible. A Network Interdiction Tool has been developed with commercial and DoD software to utilize integer programming optimization techniques to determine an optimal interdiction strategy for systems which have a network architecture. Other analytical tools have been integrated with the optimization routine to provide quantitative comparisons of targeting strategies under varying assumptions and objectives. Due to inherent limitations of any model, it is paramount that the reasoning power of the expert operational planner (or "strategist") be maintained "in the loop." This has been achieved by the development of a robust graphical user interface.

INTRODUCTION

The Network Interdiction Tool (NIT) has application to numerous complex systems which are of tactical and strategic interest. The common element among such systems is the ability to model their functionality as a capacitated flow network. Such a model can be used to determine the most efficient allocation of weapons for specified mission requirements. The fundamental measure of an allocation's efficiency is the number of weapons required to achieve a specified reduction in the steady state capacity for flow between critical points in the network.

Weapon allocation against complex networks is often determined by tedious application of manual analysis. System configurations are contained on hundreds or even thousands of hand-annotated hard copy charts. Electronic databases exist for targetable facilities, however the network connectivity of these facilities is not generally available on such databases. Manual analysis is very time consuming, but can yield an effective allocation to a simple objective when complete elimination of all

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flow between critical points is required. However, when multiple simultaneous objectives must be evaluated, or when the goal is reduction rather than complete elimination of flow, manual analysis quickly becomes overwhelming.

It is the intent of NIT to assist the operational planner in selecting target sets in three fundamental ways: (1) provide a graphic interface to allow examination of the network structure and facilitate the specification of assumptions and objectives, (2) provide an automated routine to select the "optimal" target set, and (3) provide post-solution analysis to improve understanding of solution sensitivities.

This paper will describe the fundamental modeling concepts of NIT, provide interdiction examples with a notional network, discuss additional analysis tools, and provide some detail on NIT implementation at USSTRATCOM.

MODEL

To keep this paper unclassified, references to specific systems or types of systems have been omitted and a notional network is postulated. This notional network will be examined as an undirected and capacitated network. In the basic model it is assumed that every target requires exactly one weapon. It is also assumed that allocating a weapon to any target on an arc will, with certainty, completely eliminate flow on that arc. These traits are common to some application systems and are selected to demonstrate the capabilities of NIT under the most straight forward circumstances. Other conditions and assumptions will be addressed briefly in the section labeled "EXTENSIONS."

The problem can be easily formulated to target either arcs or nodes of the network. In some applications nodes are major facilities in the system which may be appropriate targets. In other applications the "targetable" facilities lie along the arcs, and the nodes are simple junction points with little or no target value. In most applications there is some combination of arc and node targets, in this case it is relatively simple to construct artificial arcs to model all targets as lying on arcs. In this paper it will be assumed that all targets lie along existing arcs in the model network.

Integer Program Formulation

The objective of the Integer Program (IP) is to select the minimum number of targeted arcs to meet flow capacity reductions. The essential structure of the IP is derived from formulations developed by R. K. Wood [1] and will be discussed briefly for completeness.

The IP examines cut sets between the source and sink nodes and determines the minimum number of targetable arcs spanning the cut which would have to be broken in order for the remaining arcs of the cut to meet the user-specified flow capacity limit. The minimum set of such targetable arcs found is the optimal set of target arcs. A specific target may then be chosen for each arc. The formulation for a single objective scenario is:

Minimize: $\sum_{arcs} \gamma_{ij}$ (1)

Subject to:

$\alpha_i - \alpha_j + \beta_{ij} + \gamma_{ij} \geq 0$ $\forall$ arcs (2a)

$\alpha_j - \alpha_i + \beta_{ij} + \gamma_{ij} \geq 0$ $\forall$ arcs (2b)

$\sum_{arcs} u_{ij}\beta_{ij} \leq G$ (3)

$\alpha_i = 1$ $\forall$ sinks (4a)

$\alpha_i = 0$ $\forall$ sources (4b)

$\alpha_i, \beta_{ij}, \gamma_{ij} \in \{0,1\}$

Variables:

$$\alpha_i \equiv \begin{cases} 1 & \text{if node } i \text{ is on the SINK side of the current cut} \\ 0 & \text{if node } i \text{ is on the SOURCE side of the current cut} \end{cases}$$

$$\beta_{ij} \equiv \begin{cases} 1 & \text{if arc } ij \text{ spans the cut, but is NOT TARGETED} \\ 0 & \text{otherwise} \end{cases}$$

$$\gamma_{ij} \equiv \begin{cases} 1 & \text{if arc } ij \text{ is TARGETED} \\ 0 & \text{otherwise} \end{cases}$$

Constants:

$u_{ij} \equiv$ capacity of arc ij

$G \equiv$ flow capacity goal between source(s) and sink(s)

The objective equation (1) minimizes the number of arcs broken (selected for targeting). Constraints (2a) and (2b) require that any arcs which span the cut under consideration ($|\alpha_i - \alpha_j| = 1$) be designated as targeted ($\gamma_{ij} = 1$) or untargeted ($\beta_{ij} = 1$). Directed networks can be accommodated by omitting constraint (2a) or (2b). Constraint (3) requires that spanning arcs which are not broken have a combined capacity of no more than the flow capacity goal. Constraints (4a) and (4b) simply specify which nodes are designated as sources and sinks, which must always be on their respective sides of the cut. If N is the number of nodes in the network and A is the number of arcs, this formulation will have N+2A binary variables.

Some arcs will not contain any appropriate target. There are a number of ways to specify in the IP formulation that some arcs in the network are untargetable. The simplest way is to add a set of constraints:

$\gamma_{ij} = 0$ $\forall$ untargetable arcs

These constraints are very simple to build and their "untidiness" is eliminated with substitutions in the preprocessor of the IP solver.

Multiple Objective Modification

To expand the formulation for solving multiple sets of objectives, each with their own flow capacity goals, the formulation is modified slightly. Now a simultaneous cut is examined for each objective. This requires that most variables and constants pick up an additional index "k" to track each objective. The γ_{ij} variables do not require the additional index, since if an arc is targeted for any objective, it is always targeted. In this paper it will be assumed that arc capacities are constant for each objective, therefore the additional index is not assigned to the u_{ij} constants.

Constraints must be repeated for each objective. The resulting formulation for multiple objectives is:

$$
\begin{array}{ll}
\text{Minimize:} \quad \sum_{\text{arcs}} \gamma_{ij} & \\
\text{Subject to:} & \\
\alpha_{ik} - \alpha_{jk} + \beta_{ijk} + \gamma_{ij} \geq 0 & \forall \text{ arcs}, k \\
\alpha_{jk} - \alpha_{ik} + \beta_{ijk} + \gamma_{ij} \geq 0 & \forall \text{ arcs}, k \\
\sum_{arcs} u_{ij} \beta_{ijk} \leq G_k & \forall\, k \\
\alpha_{ik} = 0 & \forall \text{ sinks}, k \\
\alpha_{ik} = 1 & \forall \text{ sources}, k \\
\alpha_{ik}, \beta_{ijk}, \gamma_{ij} \in \{0,1\} &
\end{array}
$$

If K is the number of objectives in the scenario evaluated, with N and A as defined previously, this formulation contains K(N+A)+A binary variables. For a typical targeting application there may be 1000 nodes, 1500 arcs, and 5 objectives, yielding 14,000 variables.

EXAMPLES

Network Characteristics

The primary data structure for the network is an arc list. Each arc entry contains flow capacity data and is linked to its end-nodes in a node database. An available target database is maintained with links to associated arcs in the arc list. The essential structure of these files is shown in Figure 1. The designation of nodes as "from" or "to" in the Arc List file is arbitrary for this undirected network. Dual linkage between lists is beneficial, e.g. the Arc List file may contain a pointer to a link list of targets associated with that arc.

The throughput potential for each arc is examined through intelligence sources and a maximum flow capacity is assigned. If the strategist is only interested in objectives in which flow capacity is completely eliminated between critical nodes, these capacity assignments are irrelevant. If, on the other hand, the strategist is interested in partial reductions in flow capacity, the solutions may be very sensitive to individual arc capacities.

Targets are categorized by applicable attributes, which may include geographic location relative to political borders (international or internal), weapon type requirement, repair time, defenses, or potential collateral damage effects. Categories of targets can then be included or excluded from targeting consideration based on interdiction strategy and assumptions.

Figure 2 shows the notional network annotated with arbitrarily assigned arc capacities and target locations for each arc. Three categories of targets are indicated on Figure 2 by labeling targets A, B, or C.

Scenario Specification

Establishing a solution scenario requires two specifications by the strategist: the processes to be interdicted must be identified by selecting the critical points (nodes) between which product must flow, and then targeting requirements and restrictions to be employed in the scenario are specified. These specifications are made via a Graphic User Interface (GUI).

ARC LIST FILE

Arc Number	From Node	To Node	Capacity	Other Arc Data	Other Arc Data
1	1	2	2	-	-
2	1	3	7	-	-
3	1	6	6	-	-
4	2	3	1	-	-

NODE LIST FILE

Node Number	Other Node Data	Other Node Data	Other Node Data	Other Node Data	Other Node Data
1	-	-	-	-	-
2	-	-	-	-	-
3	-	-	-	-	-
4	-	-	-	-	-

TARGET LIST FILE

Target Number	Resident on Arc	Target Type	Other Tgt Data	Other Tgt Data	Other Tgt Data
1	1	A	-	-	-
2	1	A	-	-	-
3	1	B	-	-	-
4	2	A	-	-	-

Figure 1. Data Structure

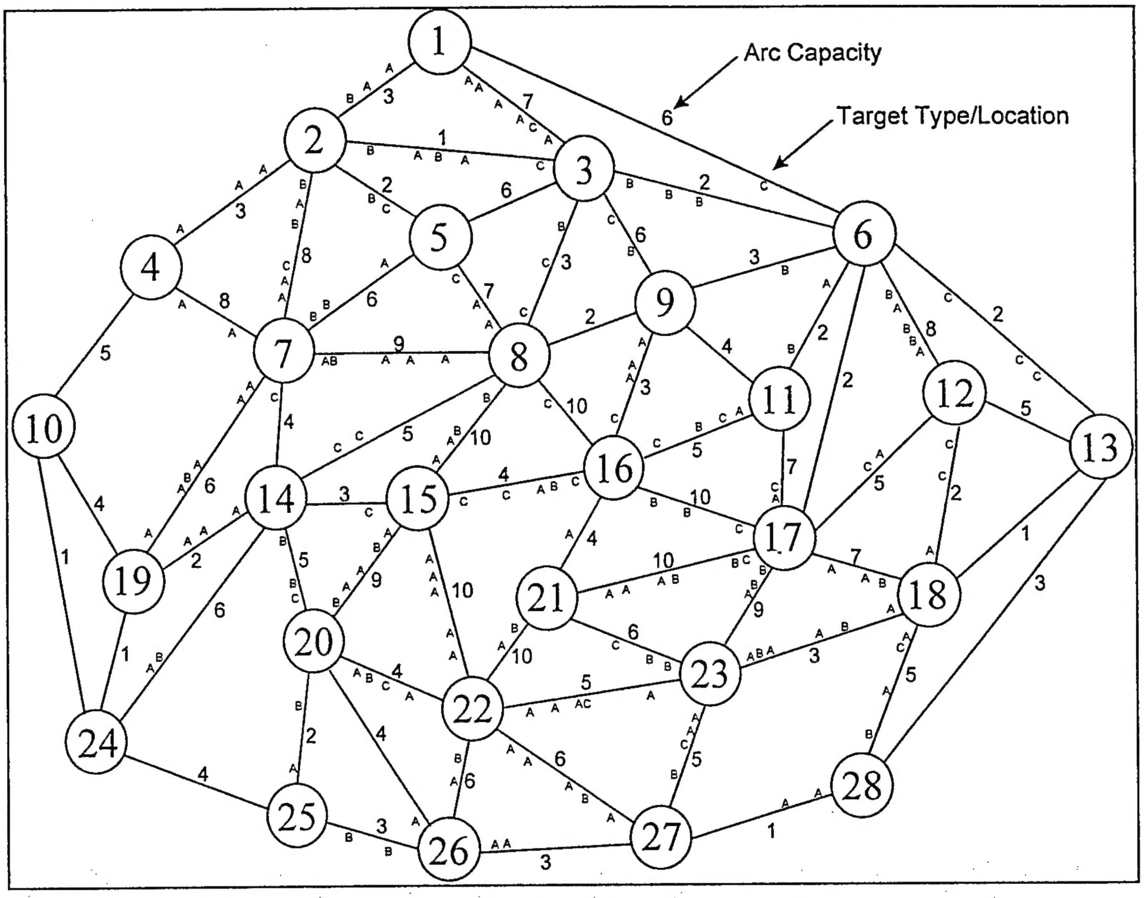

Figure 2. Notional Network

The objective of interdicting flow networks is to disrupt critical processes which require flow in the network between certain locations. It is desired to eliminate all flow capacity between these locations, or at least to reduce the capacity to some specified level. The origin and destination locations (nodes) for these steps will be referred to as the source and sink nodes for a particular objective. Objectives may have multiple source and/or sink locations. The model assumes that for a single objective, it is desired to interdict flow from any source node to every sink node. Multiple independent processes may be specified, each with their own source/sink combinations.

Target characteristics required for the interdiction mission are determined by rules of engagement, weapon availability, mission goals, and other considerations. The strategist's requirement may be to interdict the system for a specified period of time, so the strategist specifies only targets with repair times meeting this requirement. There may be limitations on the degree of collateral damage risked, in which case only targets with low collateral damage potential are activated. Once these requirements are specified, the target database is interrogated to generate a list of qualified targets. Only arcs which contain qualified targets are allowed to be selected in the solution set. The GUI will visually indicate which arcs contain qualified targets.

Notional Applications

Figure 3 illustrates Scenario 1. This is a simple scenario in which a single objective has been defined with node 8 as a

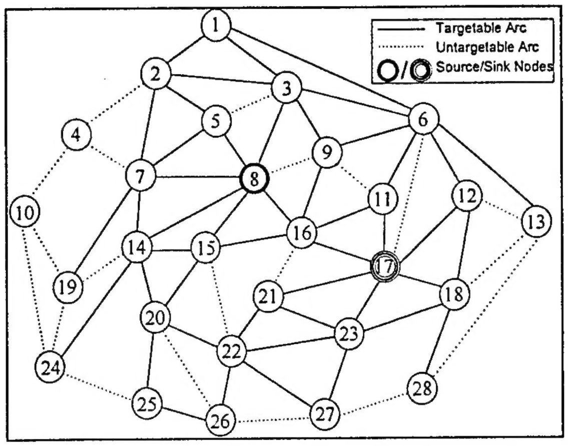

Figure 3. Scenario 1

source, and node 17 as a sink. Arcs containing target types B and C have been activated for targeting. Arcs containing only A type targets or no targets at all may still carry flow, but cannot be selected for targeting.

Once the objectives and assumptions have been entered, NIT writes the IP formulation to a file and directs the IP to be solved. A single objective solved for an actual network application with several hundred or a few thousand arcs and nodes generally takes only a few seconds to solve. NIT solves Scenario 1 in a fraction of a second. Once the solution is found, it is displayed by highlighting the arcs selected for targeting. The solution displayed for this scenario is shown in Figure 4. Available target types and capacities for each arc are also shown. Note that arc 8-9 cannot be selected because it has no targets. Once the optimal arcs are selected by the IP, the strategist uses the GUI to examine the arcs selected for targeting and determines which specific target should be attacked to cut flow across each arc. For scenario 1, the strategist would choose one B or C type target on each of the twelve arcs selected by the IP. This selection process may be dependent on numerous secondary characteristics and considerations which are best left to the logic of the strategist.

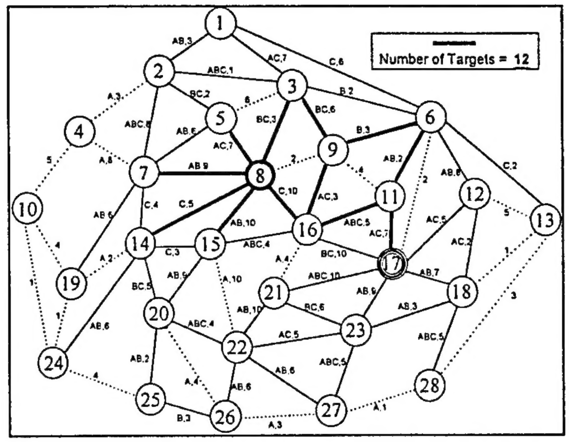

Figure 4. Scenario 1 Solution

The strategist may alter the assumptions of the scenario and re-examine the optimal target set. For example, if the targeting of type A targets were allowed, fewer weapons may be required. Figure 5 displays the solution when arcs containing A, B or C type targets are allowed. A combination of ten arcs is selected. The decision can now be made whether this savings of two weapons over the previous solution is worth the cost of reduced target desirability. It should be noted that only one type A target, on arc 27-28, is required to be attacked for the new set.

Intelligence sources have not necessarily located every potential target in a network. By solving the IP allowing any arc to be targeted, even those without a known

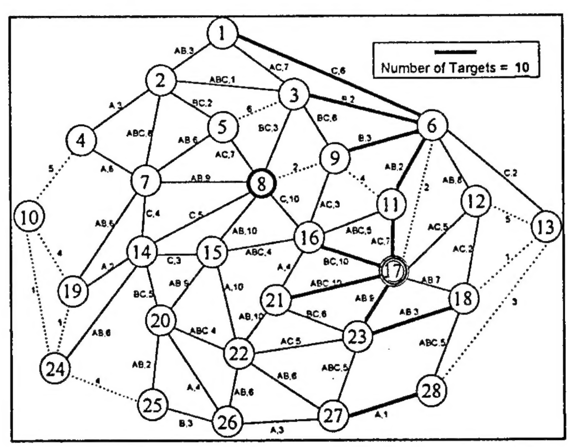

Figure 5. Scenario 1, All Target Types

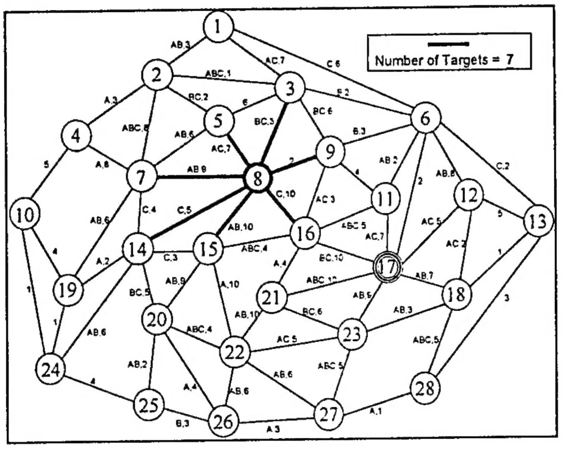

Figure 6. Scenario 1, All Arcs

target available, a minimum theoretical set of target arcs can be found. These arcs can then be rigorously examined for the desired target types. Figure 6 illustrates this solution set, containing seven arcs. Comparing Figure 6 with Figure 4, it is evident that if a B or C type target could be identified on Arc 8-9, a savings of five weapons could be realized in the original Scenario 1 mission.

Partial Interdiction Strategies

Another approach to reducing the weapon requirements for a particular interdiction is to evaluate the flow reduction requirement. NIT has the ability to run a simple maximum flow linear program (MFLP) which finds a maximum initial flow between the Scenario 1 source and sink of 46 units. In the examples above, the final flow between objective nodes was required to be zero. If it is acceptable for one unit of flow to remain after interdiction (approximately a 98% interdiction), the strategist can maintain the original targeting requirement of types 'B' and 'C' and still reduce the weapon requirements by three weapons as shown in Figure 7. Additional savings are realized as the allowable flow level is raised. If five units of flow are allowed (approximately 89% interdiction), only five weapons are required.

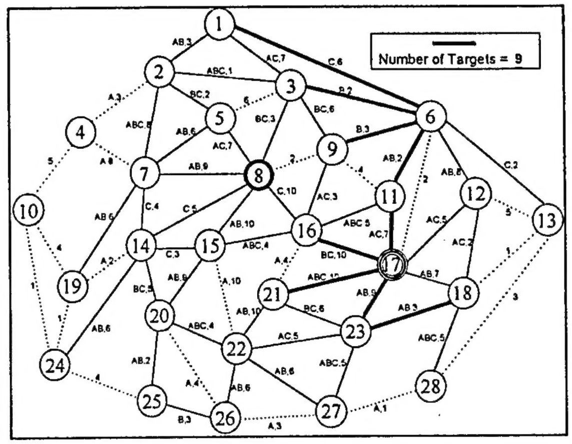

Figure 7. Scenario 1, 98% Interdiction

A critical point that must be made is that partial interdiction solutions (i.e. relaxation of constraint (3)) can require significantly longer IP solve times than full interdiction. In an actual network application, the solve time for complete interdiction was less than one minute. The solve time with approximately 90% interdiction was 1.5 hours.

Multiple Objectives

Manual target selection becomes more difficult when multiple objectives are introduced. Scenario 2 involves the original objective (source/sink of 8/17) with one additional objective (source/sink of 14/28). Figure 8 shows the optimal arc set for Scenario 2 under complete interdiction with target types B and C active. Also shown is a combined cut set for the two objectives. The circuitous nature of this cut set is typical of multiple objective interdictions. Frequently, cut sets are disjoint among multiple objectives. Figure 9 shows the solution to Scenario 3 which contains four objectives. Two cuts are shown in Figure 9. One cut satisfies three objectives, and the second cut satisfies the other objective. This pair of cuts is not unique for the solution, but does demonstrate the increased complexity of multiple objectives. In general, more complex objective sets tend to yield less

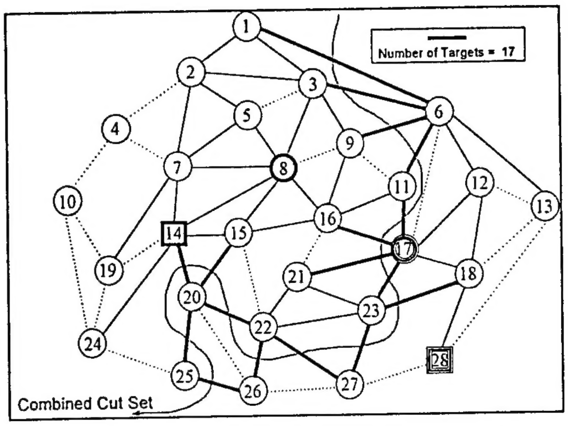

Figure 8. Scenario 2 Solution

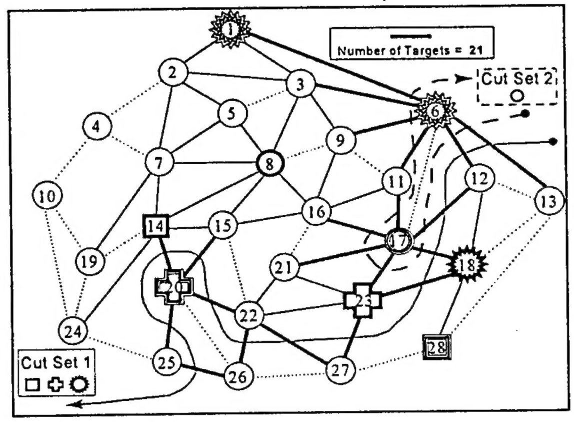

Figure 9. Scenario 3 Solution

intuitive optimal target sets. This fact will be further demonstrated later in the section labeled "IMPLEMENTATION."

EXTENSIONS

Many questions and issues remain after the "optimal" solution is displayed. Several facilities are being developed for NIT to help quantify effects and answer questions for the strategist. Two of the most significant facilities will be discussed.

Targeting Efficiency

As seen earlier, modest reductions in interdiction level requirements can significantly reduce weapon requirements. It would be useful for the strategist to have a graphical representation of a range of possible interdiction levels. If a small number of weapons are to be applied across several objectives or several networks, the strategist may want to apply additional weapons only so long as significant capacity reductions are achieved for each additional weapon allocated. For these circumstances it is valuable to have some measure of targeting efficiency.

NIT is capable of determining targeting efficiency by multiple IP solves at varying flow capacity goals. The strategist must specify which goals to vary and by what degree and increment. For multiple objectives, goals may be varied one objective at a time, or all goals may be varied simultaneously. Figure 10 shows the results from relaxation of the interdiction of each Scenario 3 objective independently. This analysis indicates that a five percent relaxation of objective 1, while still requiring complete interdiction for the other three objectives, will reduce the weapon requirement by four. Independent relaxation of objectives 2 and 4 has no benefit.

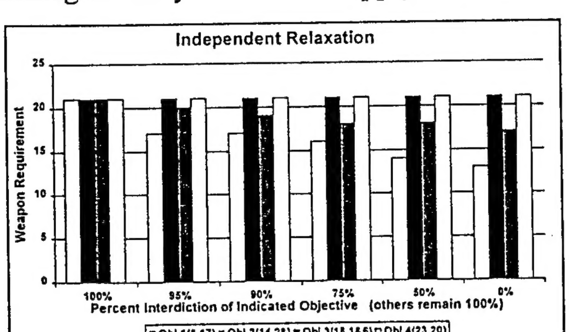

Figure 10. Independent Relaxation

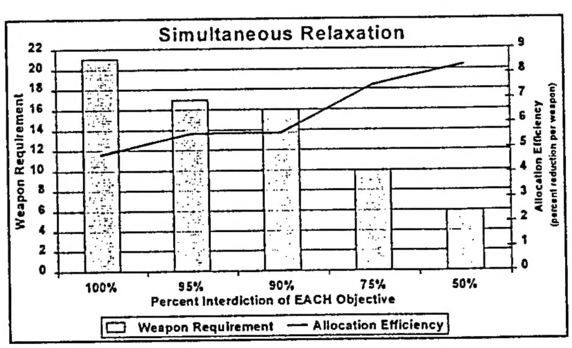

Figure 11. Simultaneous Relaxation

Figure 11 shows the result of simultaneous relaxation of all goals by the same degree. Also shown in Figure 11 is a measure of allocation efficiency expressed as percent interdiction per targeted arc. Even though partial interdictions are sometimes not tolerable by mission needs, it is helpful to understand the dynamics of efficiency.

Risk Assessment

The IP used in NIT assumes that every weapon is successfully employed to completely break the target arc. This assumption is clearly over optimistic with any weapon system. Even so, in many applications, the weapon reliability is great enough and the cost of redundant targeting is high enough that it is still preferred to select targets based on this deterministic assumption. It is beneficial, nonetheless, to quantify the mission risk associated with the interdiction strategy developed.

A probability of successful attack can be determined or assumed for each attack based on weapon system reliability, target characteristics, and other planning factors. NIT will run simple Monte Carlo determinations of the success of planned attacks. For each run, a simple MFLP is run to determine the resulting flow capacity for each objective. Statistics are gathered on multiple runs to determine a distribution of possible outcomes to the full attack. As an example, consider the attack for Scenario 1 with full interdiction shown in Figure 4. Assume that type C targets are preferable to type B targets, so a target of type C will be selected for any arc containing both types. The probability of a successful attack is arbitrarily assigned as 0.8 and 0.9 for all B and C type targets respectively. The result of the interdiction risk analysis is shown in Figure 12. Because this tool is not yet fully automated in NIT, only 100 runs were

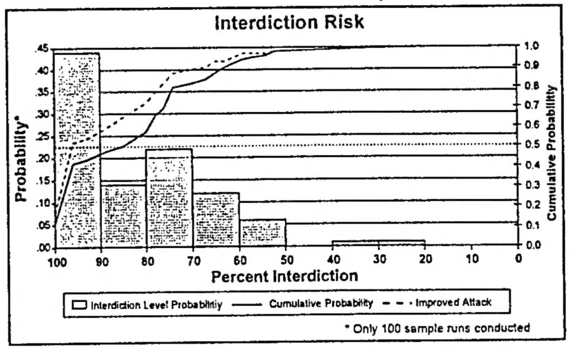

Figure 12. Scenario 1 Interdiction Risk

conducted to obtain data for this analysis. A more statistically significant sample size will be produced when NIT performs automated runs.

As shown in Figure 12, there is a probability of roughly 0.44 that an interdiction of at least 90% will be achieved for this attack. There also appears to be a probability of about 0.02 that the resulting interdiction will be less than 50%. These summary statistics must be considered with caution as they assume that each attack occurs completely independently. This is often a far from perfect assumption when attacks may have common weapon systems, delivery platforms, defensive threats, etc. A more sophisticated simulation run may be developed to more accurately accommodate such dependencies once mission planning details are completed and dependence relationships are known.

Identification of Critical Arcs

One additional benefit of conducting the risk assessment runs is that arcs which are especially critical to the interdiction strategy can be determined. Specifically, particularly poor Monte Carlo run results should not become the results of an actual mission execution. By collecting statistics on these poor performing runs, arcs which are most responsible for their occurrence can be identified. Such arcs are generally characterized by a combination of relatively poor probability of successful attack, large

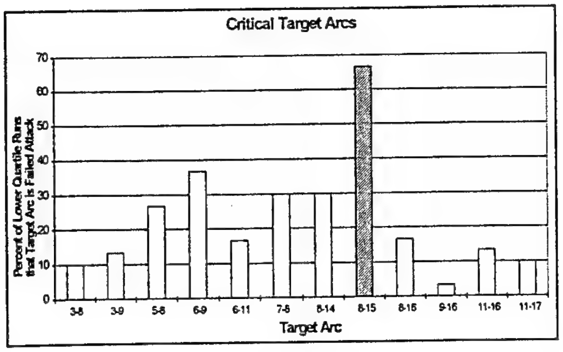

Figure 13. Scenario 1 Critical Arcs

capacity, and few bypass routes. Statistics are collected on the bottom performing quartile of the Monte Carlo runs. The frequency with which each arc occurs as a failed attack in these runs is plotted. Figure 13 shows the results of this analysis for Scenario 1. Not surprisingly, arc 8-15 is singularly significant for this interdiction due to the less desirable target type and the arc's large capacity.

Many actions may be taken to reduce overall risk by improving the probability of attack success on critical arcs. The strategist may choose to apply a more suitable weapon type, plan a second weapon to another target on the same arc, or even apply another weapon to the same target. In this example, if the strategist were to apply an additional weapon (assumed independent of the first) with the same probability of success as the first (0.8) to arc 8-15, the resultant probability of breaking arc 8-15 is 0.96. The dashed line on Figure 12 shows the resulting reduction in interdiction risk with the same 100 Monte Carlo runs used before. The probability of the interdiction level exceeding 90% has improved to well over 50%. The median interdiction level has improved from 85% to 96%.

Alternate Formulation

If there are a fixed number of weapons to apply to a given objective, it may be beneficial to reformulate the IP slightly. By essentially exchanging constraint (3) with objective function (1), the specified number of weapons will be allocated as best possible to minimize the flow capacity. This formulation is available in NIT, however was not selected as the primary formulation due to difficulties in handling multiple objectives. A flow requirement can no longer be specified for each objective. Instead, each of the multiple objectives must be assigned a relative weight if all flow is not eliminated. This makes the distribution of weapons among competing objectives less prescribeable by the strategist.

This alternate formulation does, in some circumstances, exhibit somewhat faster solve times as all non-unity coefficients are removed from the constraint equations. Additionally, the alternate formulation can be used in single objective, partial interdiction runs to resolve alternate optimal solutions. When several combinations of the same number of arcs which meet interdiction requirements exist (alternate optimal solutions), the particular combination selected by the IP may not reduce flow capacity by as much as other feasible combinations. The minimum combination which also allows the least flow may be found by executing the alternate formulation and specifying the minimum number of targets previously determined to be optimal.

Variable Weapon Requirements

In many applications it is desirable to assign different costs for targeting different arcs. This requires non-unity coefficients on the γ_{ij} variables. Application exists for this when it is desired to differentiate the cost of selecting different target types. In one application, it may require additional weapons or more valuable weapons to target different facilities. In another application, some target types may hold less utility than

others from a mission requirements perspective. In this case, rather than specifying that a particular type is completely untargetable as has been done for the examples in this paper, a higher cost coefficient is given to assigning a weapon to it, discouraging its selection. The relative value of the coefficients may be selected such that there is a one-for-one preference for particular types, or dramatically different weights could be assigned such that certain target types would not be selected at all unless no possible combination of the more desired types can meet interdiction requirements.

Variable Target Effects

In some instances, successful attack on a particular target on an arc does not prohibit all flow through the arc. For example, there may be an inherent bypass capability for the flow in an arc which is not effected by targeting. Partial interruptions may be modeled by modifying constraint (3) as follows:

$$\sum_{arcs} \left(u_{ij}\beta_{ij} + v_{ij}\gamma_{ij}\right) \le G,$$

where v_{ij} is the capacity remaining when arc ij is targeted.

In some applications, however, the relationship is not so simple. The targeted capacity is dependent on whether certain other arcs are also targeted. For example, targeting an arc may have one effect if targeted alone, and a more severe effect if an adjacent arc is also targeted. Such phenomena could be handled by introducing an additional series of variables and lookup tables for adjacent arcs, however NIT's approach is to introduce artificial arcs to the network and associated constraint equations as described below.

A sub-network is assumed, consisting of two consecutive arcs, AB and BC, as shown in Figure 14. Arcs have

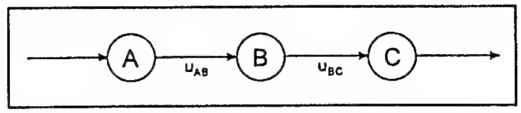

Figure 14. Sub Network

untargeted capacities of u_{AB} and u_{BC}. For simplicity, a directed network with no branching at A, B, or C is used.

It is assumed that targeting an arc ij alone will reduce the remaining capacity to a level of v_{ij}. Targeting arc ij and the next adjacent arc will reduce the ij capacity to a level of w_{ij}. Additional consecutive arc targeting will not be considered for this example. Each arc ij is split into three parallel arcs between nodes i and j as shown in Figure 15. For arc AB there is one arc representing the capacity lost by targeting the original arc alone (ABa with capacity $u_{ABa}= u_{AB}[1-v_{AB}]$), one representing the capacity lost by targeting the original arc when the next consecutive arc is also targeted (ABb with capacity $u_{ABb}= u_{AB}[1-w_{AB}]$), and one arc which cannot be targeted (ABc with a capacity $u_{ABc}= 1-u_{ABa}- u_{ABb}$). Note that the capacity of arc ABc may be negative, but the total capacity between nodes A and B is always non-negative, as simultaneously targeting arcs ABa and ABb will not be permitted by formulation constraints introduced.

The existence of multiple arcs between a given pair of nodes requires an additional index on the β and γ variables to uniquely identify each arc. Additional constraints of type (2) are added for the artificial arcs. The formulation remains otherwise the same, with the addition of two sets of constraints to force appropriate

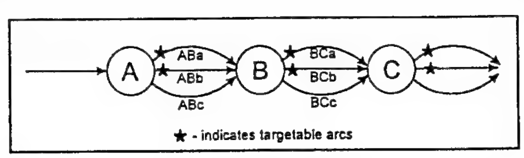

Figure 15. Modified Sub Network

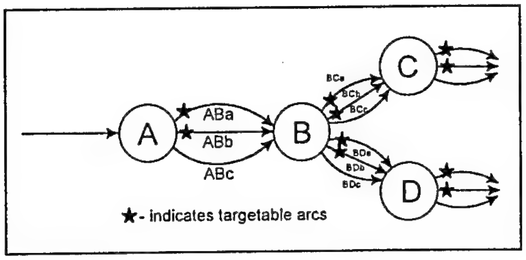

Figure 16. Branching Sub Network

targeting of the new artificial arcs. The first constraint prevents targeting the same original arc twice.

$$\gamma_{ija} + \gamma_{ijb} \le 1 \quad \forall \text{ original arcs } ij \qquad (5a)$$

The second constraint ensures that the additional reduction in capacity for targeting consecutive arcs is only allowed if the next arc in line is targeted.

$$\gamma_{j(j+1)a} - \gamma_{ijb} \ge 0 \quad \forall \text{ original arcs } ij \qquad (5b)$$

This particular construct was chosen over other possible constructs because it is able to handle branching easily. A revised sub-network with a single branching at node B is shown in Figure 16.

System characteristics at branch points vary. For some applications, the improved capacity reduction is only achieved if both subsequent branches are targeted. In other applications the capacity reduction is achieved if either of the subsequent branches is targeted. If both must be targeted, constraint (5b) is repeated for each branch. If either branch can be targeted, constraint (5b) is modified. For the above example:

$$\left(\gamma_{BCa} + \gamma_{BDa}\right) - \gamma_{ABb} \ge 0$$

IMPLEMENTATION

Software and Hardware

NIT has been developed in-house at USSTRATCOM by integration of commercial and DoD software. Several thousand lines of original C language code have been used to produce the appropriate data structures and establish integration between software packages. For example, the output file from the IP solver is parsed for key variable values. This information is then returned to the GUI to show the solution on the visual network display.

The fundamentals of this integration are not technically difficult and could be accomplished with any number of available software packages and standard hardware facilities. The specific software and hardware combination used at USSTRATCOM is shown in Table 1.

Graphical User Interface

The two essential components of the GUI are the strategist's interaction with the network structure and the specification of mission objectives and assumptions. These facilities are the mechanism by which NIT maintains the essential reasoning power of the human strategist "in the loop."

One of the greatest benefits of NIT is that the strategist is able to examine the structure of the network interactively, and at whatever scale is most appropriate. By enabling the strategist to shift screen positions, and change scale at the touch of a button, the juggling of multiple charts for different scales and locations is avoided. Each network component and target is point-and-click active. This allows the strategist to immediately call up any specification of any targetable facility in the network and explore alternate targeting options. Likewise, the specifications of an arc (capacity, available targets, etc.) may be

OFF-THE-SHELF SOFTWARE

Purpose/Use	Title	Source
IP/LP Solver	CPLEX	CPLEX Optimization
Mapping Tool	OILSTOCK	NSA
GUI Builder	Builder Xcessory	ICS
Data Display	MATLAB	MathWorks
Programming Code	C (Bourne Shell)	

HARDWARE

CPU - DEC4000/610 "Alpha" (160 MHz / 256 MB)
Display - Tektronix X Terminal (20 MB)

Table 1. Software and Hardware

examined. These facilities draw the strategist into the target selection process to perform essential reality checks and make reasoned strategic judgments.

A series of buttoned windows allow the strategist to specify mission objectives including the source and sink nodes of objective processes, required target characteristics, and interdiction levels. The ease with which these can be altered encourages the strategist to actively participate in solution development and refinement.

IP solution display is accomplished by highlighting the optimal arc set on the a visual display of the network structure and a short text indicating key solution elements such as the number of weapons required and remaining flow capacity. Manual selection of specific targets is possible via the GUI. Target selection could be automated by requiring the strategist to specify detailed target priority characteristics and having NIT evaluate the available targets on each selected arc against these priorities.

In addition to displaying IP solutions, NIT has facilities to determine maximum flow and shortest path LP solutions under any desired network conditions. For example, the strategist may take an IP solution, add or delete arcs selected for targeting and find how flow capacity is effected. Displays include active flow paths and shortest path routes.

Additional analysis tools described in this paper (targeting efficiency, risk assessment, and critical arc determination) will be included in NIT, including graphical output similar to figures shown.

Multiple Network Synergy

One of the eventual goals of NIT is to allow automated or semi-automated synergistic targeting of multiple networks. Some capability, including the ability to visually overlay multiple networks, exists currently. When optimal arcs have been determined for one network structure, other network systems may be overlaid, and collocated targets identified. The arcs on other networks associated with these collocated targets can then be designated by the strategist as having already been cut, and the IP could then be solved with this condition. Integration and manipulation of network target databases will be used to automate this process.

Improvements In Targeting Efficiency

To demonstrate NIT's potential improvement in targeting efficiency, trial cases were conducted using this notional network. Seven cases, shown in Table 2, were evaluated ranging from single source-sink full interdiction to four-objective partial interdictions. Manual analysis was limited to approximately 15 minutes per problem to compensate for the simplicity of this network relative to real applications.

The general technique used for manual selection was to try to cut every arc out of either the source or sink node. When an arc had no qualified target, the connecting node was then similarly isolated. This technique worked fine for a single objective (Case 1). When a second objective was added, the objectives were generally solved independently, usually yielding less optimal solutions (Cases 4-6). No systematic approach was used for partial interdictions (Cases 3 and 7).

TRIAL CASES							
Case	Target Types	Objective	Source(s)	Sink(s)	Interdiction Level [%]	Related Figure	Wpns Saved by NIT
1	B C	1	8	17	100	5	0
2	ABC	same as Case 1			100	6	2
3	B C	same as Case 1			98	8	3
4	B C	1	same as Case 1		100	10	4
		2	14	28	100		
5	B C	1	same as Case 1		100	none	4
		2	same as Case 4		100		
		3	1 & 6	18	100		
6	B C	1	same as Case 1		100	11	0
		2	same as Case 4		100		
		3	same as Case 5		100		
		4	23	20	100		
7	B C	same as Case 6			all at 50	13	2

Table 2. Manual vs. NIT Analysis

The same strategist who performed the manual analysis was introduced to NIT. With 10 minutes of introduction the strategist was able to perform basic actions with NIT to analyze the trail cases. NIT solution times ranged from two to six minutes including objective specification, IP solution, and target selection. All IP solve times were less than one second except case 7, which solved in about 20 seconds. Table 2 summarizes the improvements in weapons allocation realized by NIT over manual selection. The very simple nature of this network does not capture many of the challenges of manual target selection.

Challenges Of Critical Point Attacks

Weapon resource savings are clearly achievable through an IP-assisted selection of optimal target sets. There are, however, hazards to this process which must be recognized, and mitigated where possible.

Optimal target sets are highly sensitive to the accuracy of the network's connectivity. When partial interdictions are analyzed, optimal sets are somewhat sensitive to the accuracy of the assigned arc capacities in the network. When modeling any living system it is imperative that data is accurate and regularly verified and updated.

The resolution to which the system is modeled is critical. Many times a target appears to be located on a particular arc, but when the system is examined at higher resolution, it is found that an alternate flow route exists which severely reduces or eliminates the effectiveness of the target. It is therefore imperative that the system be modeled to include even the smallest of such details (see Figure 17).

In general, a manual analysis and allocation of weapons to an objective will assign weapons in excess of the minimum "optimal" set. This manner of allocation inherently yields some degree of redundancy. This "incidental redundancy" may, in some cases, mitigate the effects of attack failures. Efficient weapon allocations

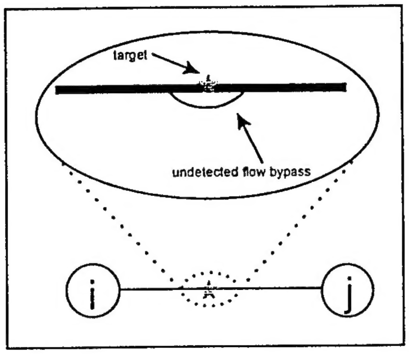

Figure 17. Network Details

determined by the IP formulation in NIT have no such accommodations. As a result, the effect of a failed attack is, in many cases, more severe to a deterministically-efficient allocation. The "Risk Assessment" facility in NIT, described earlier, is designed to quantify possible effects and reduce the impact of failures by systematically selecting where redundancy is most needed.

CONCLUSIONS

This paper described a Network Interdiction Tool (NIT) developed with off-the-shelf software to allocate weapons to capacitated flow networks. An integer programming formulation and variations were described which select optimal target sets under variable, but deterministic assumptions. Additional analytical procedures were described which provide the ability to assess targeting efficiency and manage interdiction risk. NIT was demonstrated to improve the efficiency of allocating weapons even on a very simple network structure, particularly under complex and dynamic mission objectives. NIT holds promise to provide further improvements in strategic planning by integrating attacks of multiple network structures. The author welcomes comments and suggestions for improvements to NIT.

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